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THEORY OF THE LANDING IMPACT OF SEAPLANES

By Wilhelm Pabst

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THEORY OF THE LANDING IMPACT OF SEAPLANES.*

By Wilhelm Pabst.

The present investigation is an endeavor to express the jolting stresses, designated as landing impacts, undergone by seaplanes in landing and taking off from rough water, as functions of specific factors, in order to enable the evaluation of empirically obtained results and thus acquire theoretical data for the construction of seaplane floats and hulls. A physical explanation of the landing impact, on which the following mathematical investigation is based, was given by F. Seewald,** Director of the Aerodynamic Section of the D.V.L., who first suggested the present investigation.

General Considerations on Landing Impacts

1. An ideal landing on smooth water resembles a take-off from smooth water, except that it takes place in the reversed order. The bottom of the seaplane is gradually submerged when the step approaches the smooth water surface tangentially. The increasing drag gradually reduces the speed of the seaplane, simultaneously replacing the lift of the wings by the buoyancy of

*"Theorie des Landestosses von Seeflugzeugen." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 14, 1930, pp. 217-226.
**Discussion of F. Z. Diemer's lecture on "Flugboot und Seegung," 1927 Yearbook of the W.G.L.

the water. The braking resistance consists of wave-producing and frictional resistance. Another resistance is due to the fact that, when a moving body is plunged into water, a certain mass of the water is accelerated, corresponding to the flow about the body. When the bottom plunges very quickly into the water, as may occur in alighting on rough water and meeting a wave head-on, the last-named resistance becomes so great that the first two can be neglected. Froude's and Reynolds' laws of similarity may therefore be disregarded as applied to the impacts actually considered in this connection. The following considerations prove that Newton's law of similarity does not fully apply to all cases.

2. As shown by F. Seewald, the magnitude of the impact is affected by the elasticity of the airplane, as well as by the usual factors of current methods of calculation, such as the roughness of the water, the weight of the seaplane, the landing speed and the size and shape of the float bottom. With a stiff float bottom and assumed compressibility of the water, the acceleration of a finite mass of water would take place in an infinitely short time, thus producing, according to the momentum theorem, infinitely great forces. The fact that the forces do not become infinitely great is chiefly attributable to the influence of the elasticity of the seaplane, since the compressibility of water is negligible in comparison.

This consideration is confirmed experimentally by the fact that the impact acceleration decreases from the float bottom up. This fact, which is taken care of, in the loading conditions of the D.V.L., by a higher load factor of the float, can be explained only by the effect of elasticity. The process therefore involves elastic forces in addition to inertia forces. Hence Cauchy's law of similarity is applicable in this case. It states that two phenomena are dynamically similar only when Cauchy's number $C = v\sqrt{\rho/E}$ has the same value in both cases. The fact that Cauchy's number contains the velocity v, leads directly to the surmise that the landing impact does not, or not in all cases, depend on the square of the landing speed, as would follow from Newton's law of similarity without modification. It should also be investigated as to whether, considering the effect of elasticity and the short duration of the impact, the latter is also affected by the damping action of the material.

3. The area of the float bottom coming simultaneously into contact with the surface of the water has a decisive effect on the magnitude of the landing impact. The size of this area depends on the region in which the water surface and float bottom are parallel when coming into contact with each other. These conditions are greatly affected by the roughness of the water and the manner of alighting. Certain simplifications are required by the great variety of the seaways, which change in wave shape and length according to the force and duration of the wind,

the length of the unobstructed wind path and the depth of the sea. In most cases a seaway consists of several superposed seaways. Such diagrammatic representations, as are successfully used for strength calculation in shipbuilding practice, hold good only in a very few cases but enable a mathematical estimation of the various factors, a comparison between the different models and, an improvement in the load conditions, when supplemented by experimentally determined values. Hence, the following calculations apply to the so-called "established" seaway - a seaway which, according to numerous observations,* is constant when there is a long enough unobstructed wind path, sufficient depth of water, and a steady wind of constant direction and force. The adopted wave shape is trochoidal.

Similar simplifications and assumptions should be made on the manner of landing. It would then be possible to work out landing cases for specific seaways and to calculate the landing impact by the method set forth below, using the bottom contact areas obtained from the drawing. The general conditions and the extent to which the pilot can be expected to avoid very rough landings are chiefly matters of opinion and depend moreover on the aerodynamic properties of the aircraft. Also, the seaworthiness requirements vary in the different cases according to the structural problems to be solved. Therefore, the methods of calculation presented below should be applied with discrimination in the different cases.

*Zimmerman, Von Köppen and Laas. See also Johow Foerster, "Hilfsbuch f.d. Schiffbau," 5th edition, Vol. I, pp. 416 and 420.

Among the different possibilities of alighting on rough water the maximum landing impact which a seaplane should be capable of withstanding is produced in the following case: The seaplane strikes the water flatly with the straight portion of the bottom. This may happen when, in attempting to land with the tail down, the seaplane meets an oncoming wave, or in starting, takes off prematurely and falls back on the water. The alighting moment is shown diagrammatically in Figure 1. Seaway 2 with a wave length of 11 m (36 ft.) and a wave depth of 1 m (3.28 ft.) is roughly represented in the figure. The wave is represented as usual by a trochoid. The wind speed corresponding to the seaway is approximately 3 m/s (9.8 ft./sec.). Let c_a be the seaplane speed with respect to the water. It consists of the corresponding components of the speed above ground combined with the velocity of the water. In general c_a can be replaced by the corresponding component of the landing speed normal to the keel at the maximum angle of attack. Inasmuch as the seaplane is usually brought down against the wind, the reduction in the landing speed caused by the head wind is balanced by the opposite motion of the water. Of course, the actual values may be used in any particular case.

The Accelerated Water Mass

4. As already mentioned above, all the other forces are assumed to be small in comparison with that of the impact of the

float bottom on the water. According to Lamb* the following formulas are then obtained under the sole action of the impulsive pressures of the float bottom in a two-dimensional solution, when the motion is started from the position of rest

$$u = -\frac{1}{\rho}\frac{\partial \tilde{\omega}}{\partial x}$$

$$v = -\frac{1}{\rho}\frac{\partial \tilde{\omega}}{\partial y}$$

in which $\tilde{\omega}$ is the impulsive pressure.

By means of the equation of continuity, we then obtain

$$\frac{\partial^2 \tilde{\omega}}{\partial x^2} + \frac{\partial^2 \tilde{\omega}}{\partial y^2} = 0,$$

provided ρ is constant which, on account of the slight compressibility of water, seems admissible even for very large impact forces. We shall now consider the marginal conditions for a plate of infinite length and width b lying on the water.

If the impulsive pressure of the above equations is replaced by $\tilde{\omega} = \rho\,\Phi$, in which Φ is the velocity potential, and it is considered that no impulsive pressures are exerted on the open water surface, i.e., $\tilde{\omega} = 0$ and $\Phi = 0$, the same marginal conditions prevail as for the plate of infinite length on an infinite liquid surface. The following formula is then obtained for the plate velocity at the end of the impact period, provided the one-sidedness of the process is taken into consideration.

*Lamb-Friedl, "Lehrbuch der Hydrodynamik," paragraph 12.

$$u = \frac{8\ J}{\rho\ \pi\ b^2\ \Delta\ a}$$

This applies to a plate element of width b and length Δa when $J = P\ \Delta t$. Hence the mass of water to be accelerated is

$$M_W = \rho\ \frac{\pi}{8}\ b^2\ \Delta\ a \tag{1}$$

The distribution of the impact pressures over b is elliptical. It is assumed that the plate is absolutely rigid. In practice the flow and the pressure distribution are subject to variation.

5. The assumption of an infinite length does not apply to the actual float bottom. In fact, the length of the bottom portion which strikes the water is of the same order of magnitude as the width. Since, on the assumption of an infinite plate length, the bottom width goes into the second power, while it has a smaller power in the case of a finite bottom length, the latter must be taken into consideration, in order to avoid wrong conclusions regarding the effect of the width of the hull on the impact. In this case the bottom portion concerned can also be considered as a plate in an infinite liquid, the one-sidedness of the process being taken into consideration. The water mass accelerated by such plates of a finite chine ratio was determined experimentally by means of small vibrations. When a body vibrates in a nonviscous, incompressible, infinite fluid at rest, the mass of the body is increased by the flow which de-

velops during the motion (Stokes*, Green**). In an ideal non-viscous fluid this flow is a potential flow. In viscous fluids the potential flow can be maintained with a good approximation, provided the body makes very short and quick vibrations.*** Under the above assumptions of disregarded friction and wave formation, this fact permits of easily determining the accelerated water mass as already suggested by Föttinger for other purposes but, so far as I know, never put into practice.

Figure 2 shows the test installation. Plate 1, stiffened by a longitudinal rib, is secured to a duralumin tube 2, which connected with two steel springs 3 and 4, can vibrate along its longitudinal axis. This system, which is capable of vibrating, is deflected approximately 0.2 mm (0.008 in.) and then suddenly released by severing a wire. The resulting damped vibration was plotted by means of a scratch recording device 5,**** directly and without lever transmission, with a diamond on a glass plate moved laterally by the electric motor 6. The resulting diagram was estimated under a microscope using the simultaneously recorded time marks. This estimation, made on the assumption of proportional damping, showed that the influence of the damping on the period of vibration was negligibly small.

*Stokes, "On Some Cases of Fluid Motion." Camb. Trans., 8, 1843, Math. and Phys. Papers I, p. 17.

**Green, "Researches on the Vibration of Pendulums in Fluid Media." Trans. R. S. Edin., 1883, Math. Papers, p. 315.

***Föttinger, Jahrbuch d. Schiffbautechn. Gesellschaft, 1924.

****Pabst, W., "Aufzeichnungen schneller Schwingungen nach dem Ritzverfahren," Zeitschrift des Vereines deutscher Ingenieure, 1929. No. 46.

Figure 3 is a microphotograph of such a vibration and of the corresponding time marks. Before the tests, the spring constant was determined by loading the device with known weights and recording the resulting deflection (Table I).

TABLE I. Determination of Spring Constant

No.	Load kg	Distance from base line 1/100 mm	$K = \frac{P}{f}$ kg/cm	Mean K value kg/cm
1	2.5	14.3	175.0	176.0
2	3.0	17.0	176.5	
3	3.5	20.0	175.0	
4	4.0	23.0	174.0	
5	4.5	25.5	176.5	
6	5.0	28.0	178.5	
7	5.5	31.0	177.5	
8	6.0	34.0	176.2	
9	6.5	37.0	175.9	
10	7.0	40.0	175.0	
11	7.5	42.5	176.5	

The mass of the instrument was then determined by causing it to vibrate in air. A comparison of the mass determined by vibration with that obtained by weighing showed that the steel springs participated in the vibrating mass of the device to an extent of 35.8% of their total mass. The vibration of the plates (the dimensions and weights of which are given in Table II) against water was then tested by placing the device over the water-filled tank shown in the background of Figure 2. The water surface was approximately 45 cm (18 in.) above the plate and did not seem to be affected by its vibration. At the point of immersion of the tube, a concentrically progressing undulatory motion of very small amplitude was observed. It was merely due to friction and capillary action and probably did not affect the vibration of the plate. The longer of the four plates tested were stiffened in the plane of symmetry of the flow in order to avoid natural vibrations of the plate. The plate edges were rounded off elliptically. The results are given in Table III.

TABLE II. Dimensions and Weights of Plates

Plate No.	a mm	b mm	a:b –	G g
1	100	100	1	82
2	200	100	2	247
3	300	100	3	343
4	400	100	4	460

All moved parts without plates and springs G = 170 g
Weight of springs G = 420 g

TABLE III. Results

Plate No.		T s	Total mass M $\frac{g}{cm}s^2$	Mass of device M_A $\frac{g}{cm}s^2$	Mass of water M_W $\frac{g}{cm}s^2$	$M_W\left(\frac{a}{b}\right)^3$ $\frac{g}{cm}s^2$
1	against air	0.0096	–	–	–	–
1	" water	0.01391	0.863	0.410	0.453	0.453
2	" "	0.01990	1.752	0.578	1.174	0.146
3	" "	0.02438	2.632	0.677	1.955	0.073
4	" "	0.02840	3.590	0.796	2.794	0.050

The four measured values provide seven points of the diagram in all, since the moved mass can be easily calculated for the reciprocal edge ratio. The results are plotted in Figure 4.

As was to be expected, the curve runs, for large a/b values, parallel to the line $\rho \frac{\pi}{8} b^2$ a for a plate portion of the length a of the infinitely long plate. Thus, for the finite plate length, a constant value $\rho \frac{\pi}{16} b^3$, must be deducted for the edge effect when $\frac{a}{b} \gtreqless 1$, so that

$$M_W = \frac{\pi}{8} \rho \left(a\ b^2 - \frac{b^3}{2}\right) \qquad (2)$$

For very small values the curve approaches the parabola $\rho \frac{\pi}{8} a^2 b$. Approximately within the range $\frac{a}{b} = 1 - 2$, which concerns the present problem, a material reduction of the accelerated water mass is achieved by taking the finite edge ratio into consideration.

Impact of a Flat-Bottomed Seaplane

6. The whole seaplane-float system is based on Figure 5. Let the mass M_1 of the seaplane be concentrated in one point and a spring exerting a force $P = kf$, assumed to have no mass, be fitted between the float bottom also, assumed to be without mass. A certain water mass M_2 is accelerated when the flat float bottom strikes the water.

Mass of seaplane	M_1
Mass of accelerated water	M_2
Force of spring	$P = kf$
Deflection of spring	f
Length of spring	L
Effective weight after deducting wing force	νG

Then

$$\left.\begin{aligned} M_1 \frac{d^2 x_1}{d t^2} &= kf - \nu G \\ M_2 \frac{d^2 x_2}{d t^2} &= -kf \end{aligned}\right\} \quad (3)$$

$$x_1 - x_2 = L - f.$$

For

$$\frac{d^2 x_1}{d t^2} + \frac{d^2 x_2}{d t^2} = - \frac{d^2 f}{d t^2}$$

$$\frac{d^2 f}{d t^2} + kf \left(\frac{1}{M_1} + \frac{1}{M_2}\right) - \nu g = 0.$$

The solution is

$$f = A \sin \omega t + B \cos \omega t + \frac{\nu g}{\omega^2}$$

where

$$\omega^2 = k \frac{M_2 + M_2}{M_1 M_2} = \frac{k}{\mu}$$

for

$$f = 0$$

and

$$\omega t = 0$$

whence

$$B = - \frac{\nu g}{\omega^2}.$$

The constant A results from the following consideration. Since x_1 represents the path of the C.G. of the seaplane,

$$\frac{d x_1}{dt} = c_a \quad \text{for} \quad \omega t = 0,$$

x_2 is the path of the water mass. For $\omega t = 0$

$$\frac{d x_2}{dt} = 0,$$

Hence

$$\frac{d\,f}{d\,t} = \frac{d\,x_2}{d\,t} - \frac{d\,x_1}{d\,t} = -\,c_a,$$

whence

$$A = \frac{c_a}{\omega}.$$

The solution therefore reads

$$f = \frac{c_a}{\omega}\sin\omega t - \frac{\nu\,g}{\omega^2}\cos\omega t + \frac{\nu\,g}{\omega^2}$$

$$= A\sin(\omega t - \beta) + \frac{\nu\,g}{\omega^2},$$

where

$$A = \frac{1}{\omega}\sqrt{\left(\frac{\nu\,g}{\omega}\right)^2 + c_a^2} + \frac{\nu\,g}{\omega^2}$$

and

$$\beta = \text{arc}\tan\frac{g}{c},$$

. . . .

for the maximum impact force

$$P = k\,f_{max} = A + \frac{\nu\,g}{\omega^2} = \sqrt{(\nu\,g\,\mu)^2 + c_a^2\,k\,\mu} + \nu\,g\,\mu \qquad (4)$$

where

$$\mu = \frac{M_1\,M_2}{M_1 + M_2}.$$

This formula is quite general and therefore applicable to landplanes as well, provided the mass of the wheel is disregarded. In this case $M_2 = \infty$ and the expression becomes $\mu = M_1$. When $c_a = 0$ and $v = 1$, then $P = 2\,G$, which corresponds to Poncelet's theorem of mechanics. For a seaplane M_2 is usually only a fraction of the mass.

The water mass obtained by approximate calculation for the HE 5 Heinkel monoplane corresponds to the straight portion of the flat bottom and is 20% of the mass of the seaplane. Thus the expression $\frac{M_1 M_2}{M_1 + M_2}$ in the formula is small in comparison with the expression $c_a^2 k \mu$, especially when it is recalled that the spring constant k of seaplanes is very large and that the lifting forces have not, in most cases, become zero, so that $\upsilon \ll 1$. We can therefore neglect the expression $\upsilon g \mu$ and obtain, for the landing impact,

$$P_{max} = c_a \sqrt{k \frac{M_1 M_2}{M_1 + M_2}} \text{ or } = c_a \sqrt{k M_1} \varphi ,$$

where

$$\varphi = \sqrt{\frac{w}{1 + w}} \text{ and } w = \frac{M_2}{M_1} \qquad (5)$$

On the preliminary assumption that the impact is proportional to $\lambda^2 = (l/L)^2$ where l is an arbitrary length of one and L the corresponding length of another larger and geometrically similar float, the stress $\sigma = \frac{P\,l}{W} = \frac{\lambda^3}{\lambda^3} = 1$ is therefore independent of the increase in length, a condition which should be required. For the same material and modulus of elasticity E, the spring constant k is proportional to λ. Hence, since M is proportional to λ^3, $P_{max} \sim \lambda^2$, as above temporarily assumed.

The impact is therefore proportional to the dimension of a surface and the impact load P/F is independent of the size of the seaplane, provided geometrical similarity is assumed. Moreover, the elastic impact represents a vibration of the system, its mass consisting of the bodies which strike each other. The initial conditions of the vibration are determined by the relative motion of the two bodies at the beginning of the impact.

The eccentric impact for the considered approximation is obtained in the usual way when the reduced mass is substituted for the actual mass of the seaplane

$$M' = M_1 \frac{i^2}{i^2 + r^2} \qquad (6)$$

where i is the inertia radius and

r the distance of the percussion force from the C.G.

7. The above approximation method is not quite satisfactory, since the seaplane itself is assumed to be rigid, while only the bottom is considered elastic. The same load factor for all parts is therefore obtained by calculation. As a matter of fact, the elasticity is distributed over the whole mass system, so that the seaplane portions nearest to the point of application of the impact forces, such as the floats, must withstand greater impact forces than more distant parts, such as the fuselage, wings and engines.

Yet an accurate calculation seems impossible. Therefore we must endeavor to divide the whole system into separate masses connected with each other and with the water mass by elastic members having no mass. Analytically speaking, the separation of the seaplane into two masses leads, in general, to difficult calculations, but supplies analytically simple results which make it possible to answer a number of questions. The calculation is based on the diagram in Figure 6. Assume the fuselage, with the engine, wing and tail surfaces, to be rigid and have a mass M_1 and a moment of inertia θ_1 and represent it by the line AB. The float, likewise assumed to be rigid, is given a mass M_2 and represented by the line CD. The float and fuselage are connected by two elastic members which are assumed to have no mass. The spring constants of these members are k_v and k_h. Between the float and the water mass there is also inserted an elastic member which represents the float bottom and which has a spring constant k_2. The other notations are given in Figure 4.

The manner of calculation is derived from Lagrange's equation

$$\frac{d \frac{\partial E}{\partial \dot{q}_i}}{d\,t} - \frac{\partial E}{\partial q_i} = F_i,$$

in which E is the energy of the whole system,

q_i, the coordinate of the respective C.G.,

$\dot{q}_i$, the first derivative after the time t,

F_i, the external force.

Hence

$$E = \frac{M_1}{2}\left(\frac{d\,x_1}{d\,t}\right)^2 + \frac{\theta_1}{2}\left(\frac{d\,\varphi}{d\,t}\right)^2 + \frac{M_2}{2}\left(\frac{d\,x_2}{d\,t}\right)^2 +$$

$$\frac{\theta_2}{2}\left(\frac{d\,\varphi_2}{d\,t}\right)^2 + \frac{M_3}{2}\left(\frac{d\,x_3}{d\,t}\right)^2.$$

Then, for the mass M_1, we have in the X direction

$$\frac{d \frac{\partial E}{\partial \dot{q}_i}}{d t} = M_1 \frac{d^2 x}{d t^2}, \frac{\partial E}{\partial x_1} = 0.$$

Moreover, the elastic forces are introduced as an external force, so that

$$F_i = k_v [(x_1 + a \varphi_1) - (x_2 + L_1 + a \varphi_2)] +$$
$$+ k_h [x_1 - b \varphi_1) - (x_2 + L_1 - b \varphi_2)].$$

Then

$$M_1 \ddot{x} + (k_v + k_h) f_{12} + (k_v a - k_h b) \varphi = 0,$$

when

$$x_1 - x_2 = L_1 - f_{12}$$
$$\varphi_1 - \varphi_2 = \varphi$$

Similarly, for $q = \varphi_1$,

$$\theta_1 \ddot{\varphi} + (k_v a - k_h b) f_{12} + (k_v a^2 + k_h b^2) \varphi = 0$$

and for the other masses M_2 θ_2 with their coordinates x_2 and φ_2 and M_3 with x_3. The following expressions are then obtained for the present problem

$$M_1 \frac{d^2 x_1}{d t^2} +(k_v + k_h)f_{12} + (k_v a - k_k b)\varphi_{12} = 0$$

$$\theta_1 \frac{d^2 \varphi_1}{d t^2} +(k_v a - k_h b)f_{12} + (k_v a^2 + k_h b^2)\varphi_{12} = 0$$

$$M_2 \frac{d^2 x_2}{d t^2} -(k_v + k_h)f_{12} -(k_v a - k_h b)\varphi_{12} + k_2 f_{23} = 0$$

$$\theta_2 \frac{d^2 \varphi_2}{d t^2} -(k_v a - k_h b)f_{12} -(k_v a^2 + k_h b^2)\varphi_{12} + k_2 r f_{23} = 0$$

$$M_3 \frac{d^2 x_3}{d t^2} - k_2 f_{23} = 0.$$

After a double differentiation of the formulas

$x_1 - x_2 = L - f_{12}$ and $x_2 + r\varphi_2 - x_3 = L_2 - f_{23}$ and $\varphi - \varphi_2 = \varphi$

These equations can be introduced into the system of equations for the eccentric impact of the two-mass system for any arbitrary distribution of elasticity between the two masses.

$$\left.\begin{aligned}
\frac{d^2 f_{12}}{d t^2} &= (k_\upsilon + k_h)\left(\frac{1}{M_1} + \frac{1}{M_2}\right) f_{12} + \\
&\quad + (k_\upsilon a - k_h b)\left(\frac{1}{M_1} + \frac{1}{M_2}\right)\varphi_{12} - \frac{k_2}{M_2} f_{23} \\
\frac{d^2 \varphi_{12}}{d t^2} &= (k_\upsilon a - k_h b)\left(\frac{1}{\theta_1} + \frac{1}{\theta_2}\right) f_{12} + \\
&\quad + (k_\upsilon a^2 + k_h b^2)\left(\frac{1}{\theta_1} + \frac{1}{\theta_2}\right)\varphi_{12} - \frac{k_2}{\theta_2} r f_{23} \\
\frac{d^2 f_{23}}{d t^2} &= k_2\left(\frac{1}{M_2} + \frac{r^2}{\theta_2} + \frac{1}{M_3}\right) f_{23} - \\
&\quad - \left(\frac{k_\upsilon + k_h}{M_2} + \frac{k_\upsilon a - k_h b}{\theta_2} r\right) f_{12} - \\
&\quad - \left(\frac{k_\upsilon a - k_h b}{M_2} + \frac{k_\upsilon a^2 + k_h b^2}{\theta_2} r\right)\varphi_{12}
\end{aligned}\right\} \quad (7)$$

As shown by the preliminary formulas, the problem leads to vibrations with force combinations similar to the torsional vibrations of shafts. The graphical methods of calculation used for torsional vibrations of multi-mass systems may possibly be advantageously applied to the present case. It should be taken into consideration, however, that the problem differs slightly in the present case. From the analytical viewpoint, the solution of this system of equations requires a very complicated calculation.

8. For the centric impact (step impact), $r = 0$. Moreover, the equations with

$$k_\nu = k_h = \frac{k_1}{2} \quad \text{and} \quad a = b = \frac{l}{2}$$

are simplified whence there is obtained for the impact force

$$M_1 - P_1 = k_1 \; f_{12}$$

and

$$M_2 - P_2 = k_2 \; f_{23}$$

where f_{12} and f_{23} are derived from the system

$$\left.\begin{aligned} \frac{d^2 f_{12}}{d\,t^2} + k_1 \left(\frac{1}{M_1} + \frac{1}{M_2}\right) f_{12} - \frac{k_2}{M_2} f_{23} &= 0 \\ \frac{d^2 f_{23}}{d\,t^2} + k_2 \left(\frac{1}{M_2} + \frac{1}{M_3}\right) f_{23} - \frac{k_1}{M_2} f_{12} &= 0 \end{aligned}\right\} \qquad (8)$$

or

$$\frac{d^2 f_{12}}{d\,t^2} + \alpha_1^2 \; f_{12} - \alpha_2^2 \; f_{23} = 0$$

$$\frac{d^2 f_{23}}{d\,t^2} + \alpha_3^2 \; f_{23} - \alpha_4^2 \; f_{12} = 0$$

This system can be easily solved in the usual way. Solved with respect to f_{12} and with $f_{12} = A\,e^{\omega t}$, a quadratic equation is obtained for ω^2. With $\omega = i\,\lambda$,

$$\lambda^2 = \tfrac{1}{2}\left[\alpha_1^2 + \alpha_3^2 \pm \sqrt{(\alpha_1^2 - \alpha_3^2) + 4\,\alpha_2^2\,\alpha_4^2}\right] \qquad (9)$$

and we obtain for

$$f_{12} = A_1 \sin \lambda_1 \; t + A_2 \cos \lambda_1 \; t + B_1 \sin \lambda_2 \; t + B_2 \cos \lambda_2 \; t$$

$$f_{23} = \frac{1}{\alpha_2^2} \; \frac{d^2 f_{12}}{d\,t^2} + \frac{\alpha_1^2}{\alpha_2^2} f_{12}$$

or

$$f_{23} = - A_1 \frac{\lambda_1{}^2 - \alpha_1{}^2}{\alpha_2{}^2} \sin \lambda_1 t - A_2 \frac{\lambda_1{}^2 - \alpha_1{}^2}{\alpha_2{}^2} \cos \lambda_2 t -$$

$$- B_1 \frac{\lambda_1{}^2 - \alpha_1{}^2}{\alpha_2{}^2} \sin \lambda_1 t - B_2 \frac{\lambda_2{}^2 - \alpha_1{}^2}{\alpha_2{}^2} \cos \lambda_2 t.$$

The arbitrary constants are

$$A_1 = - \frac{\alpha_2{}^2 \frac{d f_{23_0}}{d t} + (\lambda_2{}^2 - \alpha_1{}^2) \frac{d f_{12_0}}{d f}}{\lambda_1 (\lambda_1{}^2 - \lambda_2{}^2)}$$

$$A_2 = - \frac{\alpha_2{}^2 f_{23_0} + (\lambda_2{}^2 - \alpha_1{}^2) f_{12_0}}{(\lambda_1{}^2 - \lambda_2{}^2)}$$

$$B_1 = \frac{\alpha_2{}^2 \frac{d f_{23_0}}{d t} + (\lambda_1{}^2 - \alpha_1{}^2) \frac{d f_{12_0}}{d t}}{\lambda_2 (\lambda_1{}^2 - \lambda_2{}^2)}$$

$$B_2 = \frac{\alpha_2{}^2 f_{23_0} + (\lambda_1{}^2 - \alpha_1{}^2) f_{12_0}}{(\lambda_1{}^2 - \lambda_2{}^2)},$$

where

$$f_{12_0}, \quad f_{23_0}, \quad \frac{d f_{12_0}}{d t} \quad \text{and} \quad \frac{d f_{23_0}}{d t}$$

are the values for $\lambda t = 0$.

The initial conditions for $t = 0$ are

$$x_1 = L_1 + L_2 \qquad x_3 = 0 \qquad \frac{d x_1}{d t} = - ca \qquad \frac{d x_3}{d t} = 0$$

$$x_2 = L_2 \qquad \frac{d x_2}{d t} = - ca$$

when

$$x_1 - x_2 = L_1 - f_{12} \quad \text{and} \quad x_2 - x_3 = L_2 - f_{23}$$

$$f_{12_0} = 0; \quad f_{23_0} = 0; \quad \frac{d f_{12_0}}{d t} = 0; \quad \frac{d f_{23_0}}{d t} = ca.$$

Hence

$$A_2 = 0 \qquad\qquad B_2 = 0$$

$$A_1 = \frac{\alpha_2^2}{\lambda_1\,(\lambda_1^2 - \lambda_2^2)}\,c_a; \qquad B = -\frac{\alpha_2^2}{\lambda_2\,(\lambda_1^2 - \lambda_2^2)}\,c_a .$$

For

$$P_1 = k_1\,f_{12} \quad \text{and} \quad P_2 = k_2\,f_{23}$$

the forces on the hull or float are

$$P_1 = c_a\,k_1\left[\frac{\alpha_2^2}{\lambda_1\,(\lambda_1^2 - \lambda_2^2)}\,\sin\lambda_1 t - \right.$$

$$\left. - \frac{\alpha_2^2}{\lambda_2\,(\lambda_1^2 - \lambda_2^2)}\,\sin\lambda_2 t\right] \qquad (10)$$

$$P_2 = c_a\,k_2\left[-\frac{\lambda_1^2 - \alpha_1^2}{\lambda_1\,(\lambda_1^2 - \lambda_2^2)}\,\sin\lambda_1 t + \right.$$

$$\left. + \frac{\lambda_2^2 - \alpha_1^2}{\lambda_2\,(\lambda_1^2 - \lambda_2^2)}\,\sin\lambda_2 t\right] \qquad (11)$$

For a more convenient calculation, let

M = mass of the whole airplane = $M_1 + M_2$,

M_1 = mass of fuselage = rM,

M_2 = mass of float = sM,

M_3 = mass of water = wM,

k_1 = ck = elasticity between M_1 and M_2,

k_2 = ek = elasticity between M_2 and M_3,

k = $\dfrac{k_1\,k_2}{k_1 + k_2}$ total elasticity between M_1 and M_3.

Forces on the fuselage (mass M_1):

$$P_1 = c_a\sqrt{k\,M}\,(\varphi_1 \sin\lambda_1 t - \varphi_2 \sin\lambda_2 t) \qquad (12)$$

$$\varphi_1 = \frac{\frac{e\,c}{s}}{2\,B\sqrt{A + B}}\,; \qquad \varphi_2 = \frac{\frac{e\,c}{s}}{2\,B\sqrt{A - B}}$$

$$A = \frac{1}{2}\left[c\,\frac{r + s}{r\,s} + e\,\frac{s + w}{s\,w}\right]$$

$$\lambda_1 = \sqrt{\frac{k}{M}\,(A + B)}\,; \qquad \lambda_2 = \sqrt{\frac{k}{M}\,(A - B)}$$

$$B = \frac{1}{2}\sqrt{\left(c\,\frac{r + s}{r\,s} - e\,\frac{s + w}{s\,w}\right)^2 + \frac{4\,e\,c}{s^2}}\,.$$

Forces on the float (mass M_2):

$$P_2 = c_a\sqrt{k\,M}\,(-\,\psi_1 \sin\lambda_1 t + \psi_2 \sin\lambda_2 t) \qquad (13)$$

$$\psi_1 = \frac{e}{2\,B}\left(\frac{A + B - C}{\sqrt{A - B}}\right); \qquad \psi_2 = \frac{e}{2\,B}\left(\frac{A - B - C}{\sqrt{A - B}}\right)$$

$$C = c\,\frac{r + s}{r\,s}\,.$$

9. When it is too difficult to calculate the spring constants, they can be determined by vibration tests on similar models. The seaplane is elastically suspended and caused to vibrate by a rotating weight eccentrically attached to the fuselage. Then, the natural vibration number of a structural member can be easily determined by its resonance with the revolution number of the eccentric weight. From this the spring constants

can be easily determined. A mean spring constant $k = \frac{1}{F} \int k_i \, d \, F_i$ must be introduced for the elasticity of the float bottom, whence the mean bottom pressure is obtained. At the points where the spring constant of the bottom is smaller than the mean spring constant, the bottom pressure decreases, while it increases at the points where the constant is greater, namely, in the neighborhood of the bulkheads.

10. A numerical example is given in the form of a calculation for a float seaplane of the Heinkel monoplane type, e.g., the HE 5, HE 8, HE 9, etc. The elasticity between the fuselage and float is determined by a vibration test made by H. Hertel and Leiss of the Static Division of the D. V. L. on an HE 8 for the determination of wing vibrations.

The float was found to develop vibrations with a frequency of 550/min., as shown in Figure 7. In the above equations therefore, we should put $k_h = \infty$, since the axis of vibration passed through the rear suspension point of the float. However, as a first approximation, we shall use the formulas derived from the simplified assumptions which, strictly speaking, hold good only for $k_v = k_h$ and $a = b$, or for a single spring located in the line of gravity of the constant k_1.

Let k_1 be the constant of a spring mounted in the line of gravity g h and producing the same type and frequency of vibration as that of the observed vibration. For the latter

(Fig. 8) we have the approximations

$$(M_1\ b^2 + \theta_1)\ \frac{d^2\ \varphi_1}{d\ t^2} = k_\upsilon\ e\ f_\upsilon$$

$$(M_2\ b'^2 + \theta_2)\ \frac{d^2\ \varphi_2}{d\ t^2} = -\ k_\upsilon\ e\ f_\upsilon$$

in which the following notation is used.

M_1,	mass of fuselage	=	275 kg²/m
θ_1,	inertia moment of fuselage	=	825 kgm²
M_2,	mass of float	=	30 kg²/m
θ_2,	inertia moment of float	=	95 kgm²

Moreover, $b = 0.86$ and $b' = 1.33$. φ_1 and φ_2 are the angles of the fuselage and float motion; f_v and k_v the deflection and spring constant of the forward suspension. For

$$f_v = e\ (\varphi_1 - \varphi_2)$$

$$\frac{d^2\ f_\upsilon}{d\ t^2} = k_\upsilon\ e^2\ f_\upsilon\ [\frac{1}{M_1\ b^2 + \theta_1} + \frac{1}{(M_2\ b'^2 + \theta_2)}].$$

We now replace the spring constant k_v at a distance e by a spring k_1 at a distance b and obtain the angular velocity of vibration

$$\omega = \sqrt{k_1\ b^2 \left(\frac{1}{M_1\ b^2 + \theta_1} + \frac{1}{M_2\ b'^2 + \theta_2}\right)} = \frac{\pi\ 550}{30}$$

By this formula $k_1 = 574,000$.

This spring constant produces a deflection of

$$f = \frac{2700 \text{ kg}}{574000 \text{ kg/m}} = 4.7 \text{ mm}$$

under the static load of the weight of the fuselage. According to a numerical estimate, the spring constant at the bottom amounts to about $k_2/F = 1,750,000$. For a float water line area of approximately 10 m² the mean deflection would be

$$f = \frac{3000 \text{ kg}}{17500000 \text{ kg/m}} = 0.175 \text{ mm}$$

under the weight of the seaplane.

The landing case represented in Figure 1 is used for the calculation. Let the airplane speed be $V = 90$ km/h = 25 m/s, the line of flight roughly horizontal and the angle of inclination correspond to the angle of attack in leveling off (about 12°). Hence, the normal speed component is $c_a = 5.2$ m/s, the length of the bottom striking the water is approximately 1.2 m, according to Figure 1, and the spring constant of the bottom $k_2 = 3,500,000$ kg/m for a width of $b = 0.86$ m.

For
$$M = 305 \text{ kg}^2/\text{m} \qquad M_1 = 275 \text{ kg}^2/\text{m}$$

$$M_2 = 30 \qquad M_3 = 2\,\frac{\pi}{8}\left(a\,b^2 - \frac{b^3}{2}\right) = 46 \text{ kg}^2/\text{m}$$

$$k_1 = 574000\,\frac{\text{kg}}{\text{m}} \qquad k_2 = 3500000\,\frac{\text{kg}}{\text{m}}$$

$$k = \frac{k_1\,k_2}{k_1 + k_2} = 483000 \text{ kg/m},$$

(that is, $r = 0.9$; $s = 0.1$; $w = 0.15$; $c = 1.16$; $e = 7.25$), the impact on the fuselage is

$$P_1 = 17800 \sin 94\,t - 3820 \sin 450\,t$$

and the impact on the float is

$$P_2 = 36800 \sin 450\ t + 4820 \sin 94\ t.$$

As shown by the calculation, the impact is a vibrational phenomenon. For the designing of airplanes it is therefore important for the struts which, during the first moment of the impact, work in tension, to be subjected immediately to a compressive force of nearly the same magnitude. Some struts may also develop vibrations in resonance with the impact and therefore collapse prematurely. The frequencies of the impact are

$$n_1 = 15/s$$
$$n_2 = 72/s.$$

11. As was to be anticipated from the preliminary statements, the result was confined to the purely elastic impact with the impact coefficient 1. In practice the impact is damped, chiefly by the internal damping of the material and by friction in the connections, joints, etc. This damping action and its effect on the impact must be determined by tests. The question will be only briefly considered here. According to Plank*, Honda and Konno,** the damping of the material is directly proportional to the velocity of deformation, so that

$$P = k\ f + \beta \frac{d\ f}{d\ t} \tag{15}$$

If this formula were substituted for $k\ f$ in equations (7) and (8), the solution of the system for λ^2 would give a

*Plank, "Betrachtung uber dynamische Zugbeanspruchung," Zeitschrift des Vereines deutscher Ingenieure, 1912.
**Honda and Konno, Zeitschrift für angewandte Mathematik und Mechanik, 1921, p. 481.

biquadratic equation. According to the magnitude of the damping coefficient, the amplitudes thus obtained are smaller than those of undamped vibrations which, moreover, die out rather quickly. The damping coefficients must be determined experimentally. It is still uncertain whether the linear agreement is actually maintained. For plywood this assumption is not even approximately correct. For metals the damping effect seems to reach values considerably above the proportionality limit, especially for duralumin. This is apparently the reason why the permissible load can be greatly exceeded with duralumin bottoms without causing failure, but merely bulging or other permanent deformation. The above statement is based on the assumption that the permanent strength is not exceeded for the corresponding load-shifting coefficient and that the material is not impaired by corrosion. Owing to the brief duration of the process and to the damping effect above the limit of proportionality, the breaking strength seems to be much greater than could be anticipated from the calculation based on static tensile tests. Within the elastic range, which alone is of interest here, the damping seems to be negligibly small, as shown by a short test made with a vibrating duralumin plate, and does not warrant the tedious calculation. It is therefore suggested, as the best approximate way of estimating the damping of duralumin floats, to use only the maximum value of the greater vibration for the maximum impact. The reason is that although, when damping is taken into consideration, the amplitude does not differ much from that with no damping, the difference becomes apparent at a greater number of vibrations. Owing to the difference between the frequencies of the individual vibrations n_1 and n_2, the maximum value of the impact force may be approximately expressed by the sum of the maximum values of the individual vibrations. With damping we can approximately assume instead that, when the maximum deflection of the larger slow vibration is reached, the smaller but quicker vibration has already died out. Similar considerations apply to the impact force on a float. In this case the deflection of the short slow vibration remains small when the maximum value of the long, fast vibration is reached, while, for the maximum of the short slow vibration the long fast one has already largely died out. In this case the maximum value of the undamped vibration of great amplitude can be substituted, with a fair approximation, for the maximum impact. Hence, the maximum impact force of the case calculated above is $P_{1\,max} =$ 17800 kg, or the load factor $e = P/G = 5.9$ and $P_{2\,max} =$ 36800 kg or the bottom pressure

$$p = \frac{P_2 \max}{F} = \frac{36800}{2.06} = 1.8 \text{ kg/cm}^2$$

In spite of the limitation $k_v = k_h$ these formulas can be used for the approximate calculation of a whole series of problems, when mass and elasticity are properly subdivided. Thus, for float seaplanes, the impact force on the engine bearers or wing can be calculated with a fair degree of accuracy, if the rest of the fuselage and the float are considered as a single mass M_2, if a mean elasticity between fuselage and water is put for k_2 and if the corresponding values of the engine bearers or wing are substituted for M_1 and k_1. Similar calculations can be made for flying boats. A subdivision of the fuselage and wing is particularly advantageous for large flying boats, the weight of whose engines, fuel, etc., is distributed over the wings.

12. In dividing a seaplane into two masses, the system of three combined vibrations mentioned in Section 7, which is rather difficult to calculate, is obtained for the eccentric impact. However, the problem seems to be covered sufficiently when the reduced mass is substituted for the mass M of the seaplane (equation 6) in the formulas for the step impact. On the sense of the calculation developed in this connection, the formula assumes the elasticity is located between the water and float and not distributed according to the above assumptions.

13. The favorable influence of elasticity leads us to attempt a reduction of the impact by installing shock absorbers as on landing gears. This can be done to a certain extent. Yet too soft springs may easily have an effect contrary to the one desired. In taking off from a choppy sea and even from slightly rippling water, instabilities may develop under the

action of the accelerated mass of water subjected to short-period variations and owing to the small natural frequency of the system. Such instabilities, like the resonance of forced vibrations, may lead to premature failure. This probably accounts for the failure of the repeated attempts to equip float gears with shock absorbers.

The V-Shaped Bottom

14. Let us also investigate the V-shaped bottom for the case represented in Figure 1. A wedge of the length a (Fig. 9), assumed to be without mass, is connected with the mass M_1 by a spring member. This wedge penetrates into the water at the time t with the speed $\frac{d\,x_2}{d\,t}$. A water mass M_2, no longer constant but a function of the width y, corresponds to the bottom portion of a width y and a depth x_2 immersed at the time t. The force on the bottom, which is assumed to have no mass, now equals the momentum increment of this variable mass with respect to time

$$P = \frac{d\left(M\,\frac{dx}{dt}\right)}{d\,t}.$$

As in the case of equation (3), we now have

$$\left.\begin{aligned} M_1\,\frac{d^2\,x_1}{d\,t^2} &= k\,f \\ \frac{d\left(M_2\,\frac{d\,x_2}{d\,t}\right)}{d\,t} &= -\,k\,f \end{aligned}\right\} \qquad (16)$$

Moreover, $x_1 - x_2 = L - f$ and $M_2 = f(y) = f(x_2)$.

For very sharp V-bottoms and very large k values we can put $\frac{d\,x_1}{d\,t} = \frac{d\,x_2}{d\,t} = \frac{d\,x}{d\,t}$, and we then obtain

$$M_1 \frac{d^2\,x}{d\,t} = - \frac{d\left(M_2 \frac{dx}{dt}\right)}{d\,t}$$

or

$$M_1 \frac{d\,x}{d\,t} = - M_2 \frac{d\,x}{d\,t} + C.$$

The integration constant is

$$C = M_1\,c_a,$$

since in the case of $t = 0$ and $M_2 = 0$, $\frac{dx}{dt} = c_a$.

$$\frac{d\,x}{d\,t} = \frac{M_1}{M_1 + M_2}\,c_a.$$

The impact force

$$P = M_1 \frac{d^2\,x}{d\,t} = \frac{M_1{}^2\,c_a}{(M_1 + M_2)^2} \frac{d\,M}{d\,x} \frac{d\,x}{d\,t}.$$

As above, we again have

$$\frac{d\,x}{d\,t} = \frac{M_1}{M_1 + M_2}\,c_a.$$

Moreover, $\tan\frac{\alpha}{2} = \frac{dy}{dx}$, when the bottom walls are straight and make an angle of α with each other. Hence

$$P = \tan\frac{\alpha}{2} \frac{M_1{}^3}{(M_1 + M_2)^3} \frac{d\,M_2}{d\,y}.$$

For the greatest width, which, with the usual bottom shapes, is likely to produce the maximum force, we have

$$P = \tan \frac{\alpha}{2} \frac{c_a^2}{\left(1 + \frac{M_2}{M_1}\right)^3} \frac{d\,M}{d\,y}_{(y=b/2)}$$

with

$$M_2 = \rho \frac{\pi}{8} \left(a\,b^2 - \frac{b^3}{2}\right) = \frac{\pi}{2} \rho\,(a\,y^2 - y^3),$$

when $a > b$

$$\frac{d\,M}{d\,y}_{y=b/2} = \frac{\pi}{2} \rho \left(a\,b - \frac{3}{4}\,b^2\right).$$

The impact force on the hull of a flying boat or on the fuselage of a float seaplane with a sharp V-bottom is therefore

$$P = \tan \frac{\alpha}{2}\, c_a^2\, \frac{\frac{\pi}{2} \rho \left(a\,b - \frac{3}{4}\,b^2\right)^{*}}{\left(1 + \frac{M_2}{M_1}\right)^3} \tag{17}$$

when α is the keel angle,

c_a the components of the landing speed normal to the keel,

ρ the density of the water,

a and b the length and width of the bottom striking the water, a being smaller than b,

M_1 the mass of the seaplane,

$M_2 = \frac{\pi}{8} \rho \left(a\,b^2 - \frac{b^3}{2}\right)$ when $a > b$.

For twin-float or twin-hull seaplanes we would have

$$P = \tan \frac{\alpha}{2}\, c_a^2\, \frac{\pi\, \rho \left(a\,b - \frac{3}{4}\,b^2\right)}{\left(1 + \frac{M_2}{M_1}\right)^3} \tag{18}$$

*A similar formula, though on the assumption of an infinitely long plate, was developed by Von Karman in a report published in October, 1929, by the National Advisory Committee for Aeronautics (Technical Note No. 321: The Impact on Seaplane Floats during Landing).

in which

$$M_1 = \frac{\pi}{4}\rho\left(a\,b^2 - \frac{b^3}{2}\right)$$

for $a > b$.

Since, according to our assumptions, the sharp V-bottom is not affected by elasticity, the development of a load factor applicable to the whole airplane is warranted. The forces on the fuselage, as compared with its mass, are therefore smaller than the forces on the float bottom.

According to the formulas, the impact is infinitely great for $\alpha = 180^\circ$. Even with a flat keel, the impact forces are excessive, so that the elasticity must then be taken into account. The equation can be integrated numerically or graphically. As a first approximation, we might confine ourselves to determining the impact of the flat bottom, taking elasticity into account, and the impact of the sharp V-bottom by the above formulas. The impact of the flat-keeled bottom may then be approximately determined by drawing through the point $\alpha = 180^\circ$ a tangent to the curve of the sharp V-bottom.

We do not feel justified in expressing the keel of so-called wave-binding shapes (Fig. 10) by the angle α of Figure 10, as was hitherto usually done. In this case, equation (16) would have to be integrated graphically or numerically or else the impact of a flat bottom would have to be considered instead.

15. The maximum bottom pressure is exerted on the immersion

of the keel. It is then

$$p = \tan\frac{\alpha}{2}\, c_a^2\, \frac{\pi}{2}\, \rho \qquad (19)$$

for single-float or single-hull seaplanes, and

$$p = \tan\frac{\alpha}{2}\, c_a^2\, \pi\, \rho \qquad (20)$$

for twin-float seaplanes or twin-hull flying boats. The maximum stress on the bottom surface, however, is probably produced at a mean bottom pressure exerted over the whole bottom surface. This bottom pressure can be easily determined by formulas (17) and (18).

16. The eccentric impact can be calculated with the same formulas if, as above, the reduced mass $M' = M\,\frac{i^2}{i^2 + r^2}$ is substituted for M_1 (page 15).

Summary of the Results

17. Figures 11 and 12 show the results which can be theoretically anticipated for Heinkel monoplanes calculated for a geometrically similar increase of dimensions but variations in landing speed and bottom angle. Inasmuch as the impact force for flat or V-shaped bottoms is proportional to a surface, $P = c\sqrt[3]{G^2}$, can be expressed as a function of the weight. In this formula the coefficient c depends on the speed and the keel angle only. The load factor is then

$$e = \frac{P}{G} = \frac{c}{\sqrt[3]{G}}.$$

The length of the waves which the seaplane can withstand is also increased, since the contact length of the bottom surface is included in the calculation and would also have to be proportionally increased, in order to preserve the geometrical similarity.

The load factor calculated for the fuselage of the 3000-kilogram seaplane in seaway 2 was 6 g. It would be wrong, however, to conclude that a seaplane calculated with this load factor cannot resist stronger seaways, since a skilled pilot usually succeeds in avoiding the case represented in Figure 1 by a tail landing. Yet seaway 2 seems to be the limit at which a pilot can bring his plane down without special training. Besides, seaways seldom correspond to conditions which can be represented diagrammatically. However, such a representation is also used in shipbuilding practice for strength calculations and is necessary in order to obtain a basis for the calculation.

Comparison with the DVL Load Assumptions

18. The load assumptions of the D.V.L. developed from data supplied by Lewe and the experience of various companies are based on geometrical similarity. The load assumptions do not account for the influence of bottom width, mass distribution, elasticity, etc. Figure 13 shows the influence of the flying weight on the load factor, according to theory and to the D.V.L.

load assumptions.* Figure 14 illustrates the influence of the bottom angle on the impact according to the two methods of calculation. There is also plotted a point taken from Bottomley's model tests,** which is closer to the theoretical values than to those of the D.V.L. load assumptions. According to theory, the velocity is proportional to the first power for flat bottoms and to the second power for sharp V-bottoms, while it is proportional to the 1.5 power under the D.V.L. load assumptions.

According to these load assumptions the bottom pressure is calculated from the 50% greater fuselage loading. In theory the float has about twice the fuselage loading, provided it is not carrying additional loads (fuel tanks). The area over which the load is distributed is about the same in theory and according to the load assumptions. In general, it can be said that the theory is not in fundamental contradiction with the empirically developed load assumptions. One advantage of the theory over the load assumptions lies, however, in the possibility of considerably more accurate calculations and thus hitting the best compromise of the different float factors, especially as regards quick take-off and adequate strength. The theory, however, requires experimental confirmation and extension by experiments which are now under way and which will soon be reported.

*Lewe, Zeitschrift fur Flugtechnik und Motorluftschiffahrt, 1920, p. 125.
**Bottomley, "The Impact of a Model Seaplane Float on Water." British A.C.A. Reports and Memoranda No. 583 (1919).

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for Aeronautics.

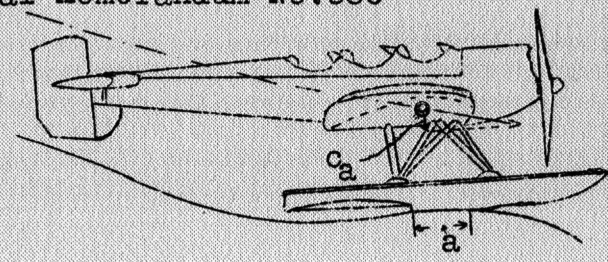

Fig.1 Seaplane alighting in seaway 2.

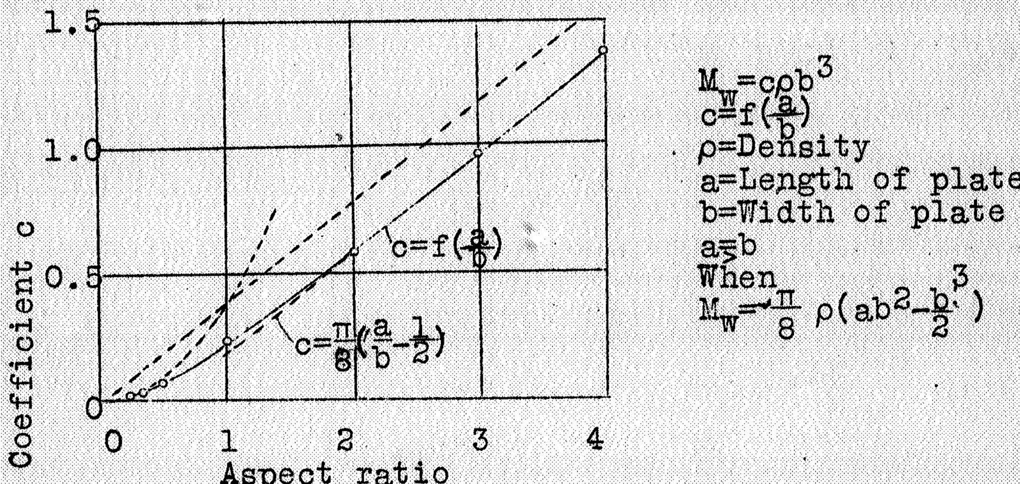

Fig.4 Accelerated water mass with one sided flow

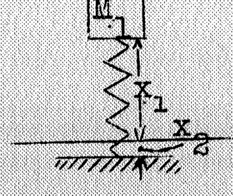

Fig.5 Diagram for the seaplane float system.

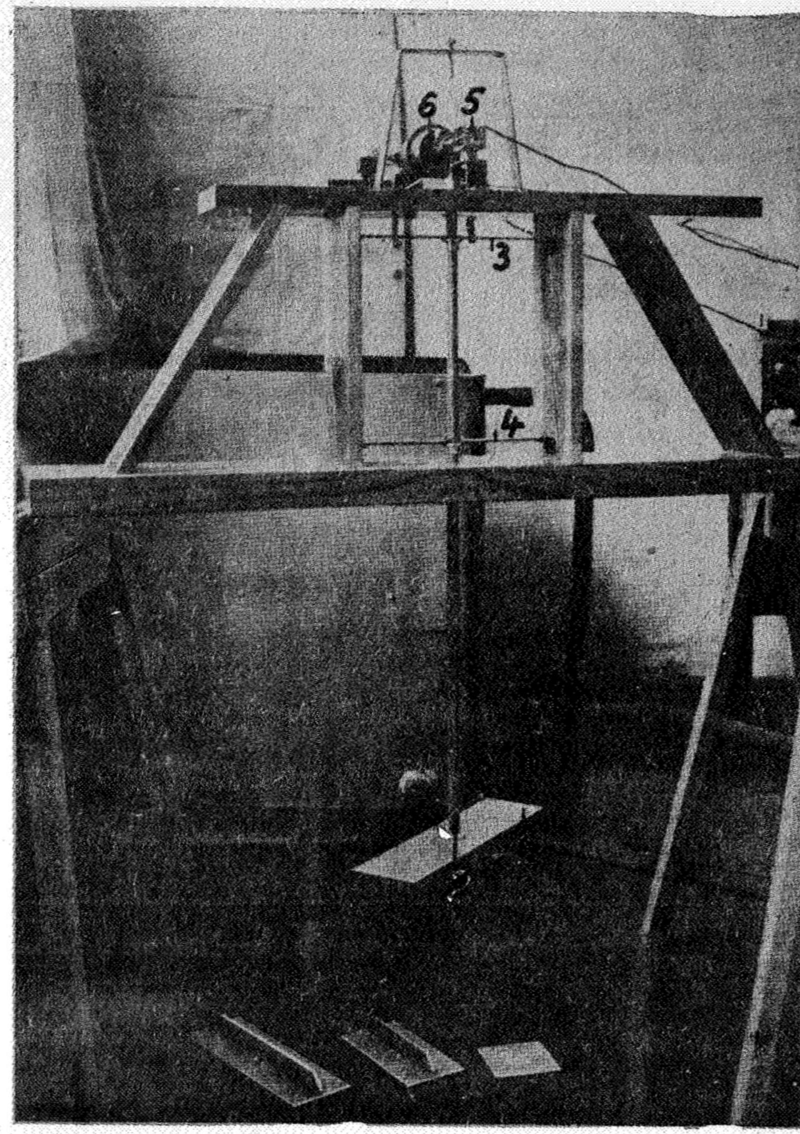

Fig.2 Test installation for the determination of the accelerated water mass.

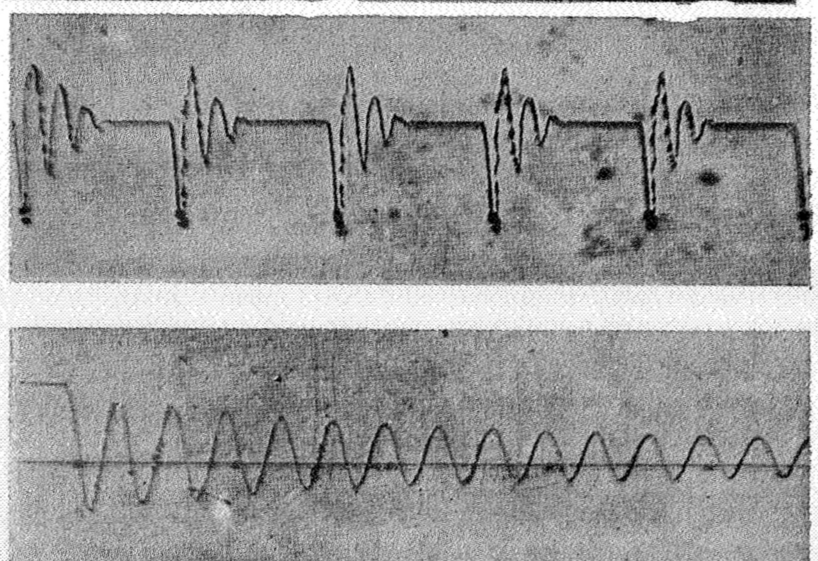

Fig.3 Microphotograph of a recorded vibration with the respective time markings.

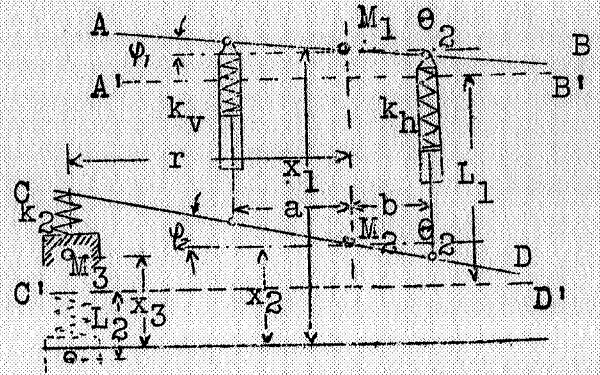

Fig.6 Diagram of eccentric impact of the two-mass system.

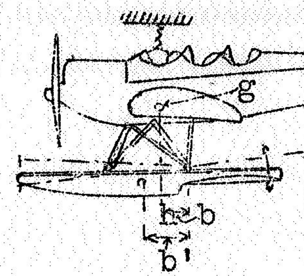

Fig.7 Vibration test for the determination of the elasticity between fuselage and float, carried out on a Heinkel HE 8 seaplane.

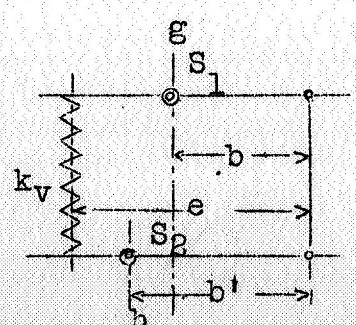

Fig.8 Diagram of the test according to Fig.7.

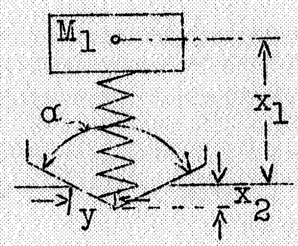

Fig.9 Diagram for V bottom tests.

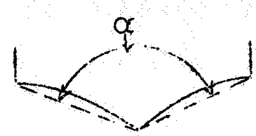

Fig.10 V bottom of wave-binding shape.

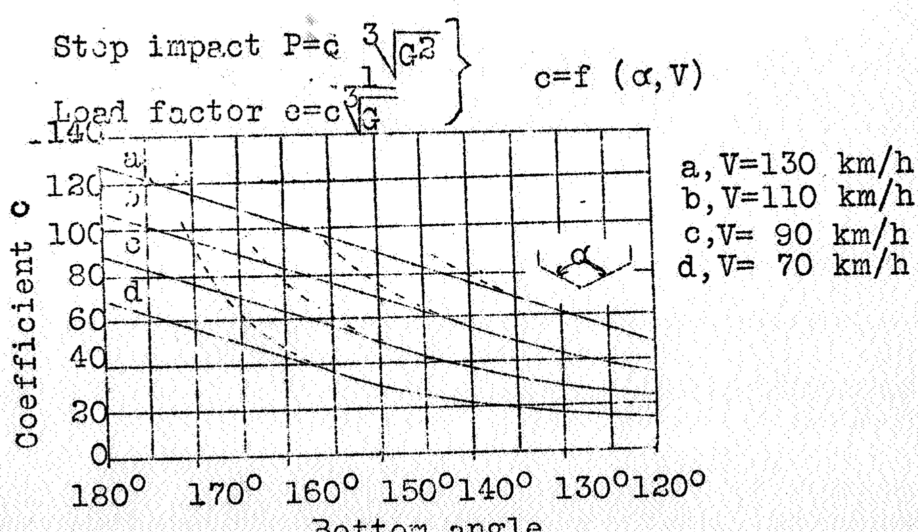

Fig.11 Alighting impact forces of twin-float seaplanes of the Heinkel monoplane type (determined theoretically).

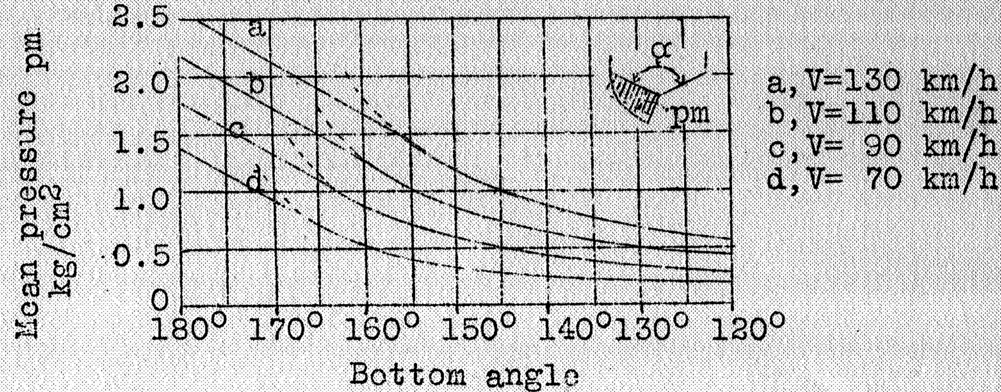

Fig.12 Water pressures at the step of twin-float seaplanes of the Heinkel monoplane type. ($p_m = f[\alpha, V]$ theoretically). The pressures are independent of the increases in size of the seaplanes.

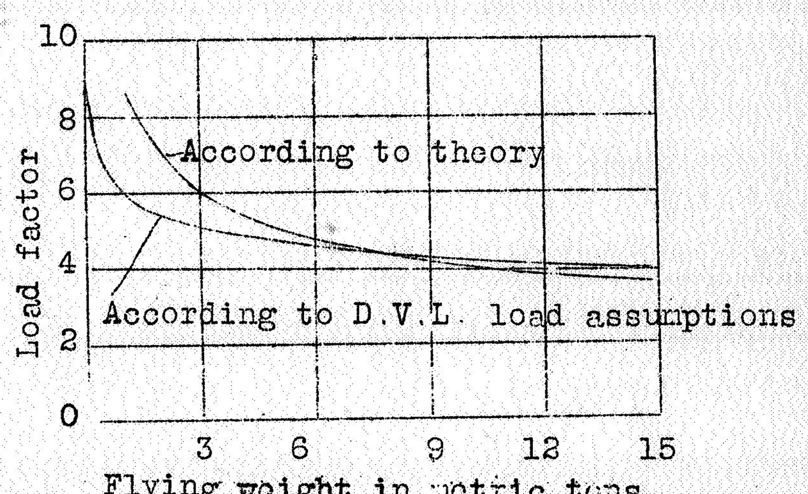

Fig.13 Relation between load factor and flying weight of similar aircraft.

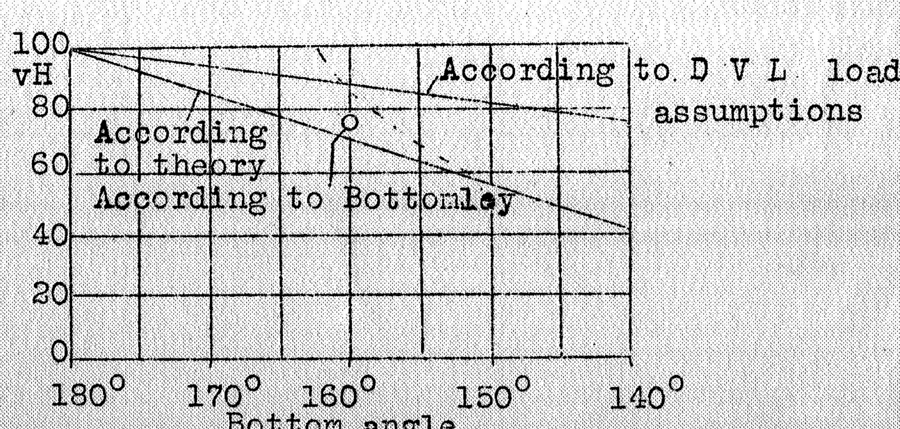

Fig.14 Impact force as a function of the V-bottom angle.